1.

2. A spin- $\frac{1}{2}$ particle of mass m moves in spherical harmonic oscillator potential $V = \frac{1}{2}m\omega^2 r^2$ and is subject to an interaction $\lambda \sigma \cdot r$. The net Hamiltonian is therefore:

$$H = H_0 + H_1$$
$$H_0 = \frac{P^2}{2m} + \frac{1}{2}m\omega^2 r^2$$
$$H_1 = \lambda \sigma \cdot r$$

- (a) What is the shift in energy for the ground state through first order in perturbation H_1 .
- (b) Compute the shift of the ground state energy through second order in the perturbation H_1 .
- 3. Let $\vec{s_1}$ and $\vec{s_2}$ be the spin operators of two spin 1/2-particles. Then $\vec{S} = \vec{s_1} + \vec{s_2}$ is the spin operator for this two-particle system.
 - (a) Consider the Hamiltonian $H_0 = \frac{1}{\hbar^2}(S_x^2 + S_y^2 S_z^2)$. Determine the eigenvalues and eigenvectors of this Hamiltonian.
 - (b) Consider the perturbation $H_1 = s_{1x} s_{2x}$. Calculate the eigenvalues of $H_0 + \lambda H_1$ in first-order perturbation theory.
- 4. Suppose that a hydrogen atom is exposed to a uniform electric field, $\vec{\varepsilon}$, and a parallel, uniform magnetic field, \vec{B} . Consider the first excited energy level, corresponding to n = 2, neglect the spin.
 - (a) Show that in general the level is split into four nondegenerate energy levels.
 - (b) For what values of ε and B are there instead only three levels, and what are the degeneracies of these levels?
 - (c) For what values of ε and B are there only two levels, and what are the degeneracies of these levels?